TAMPEREEN TEKNILLINEN KORKEAKOULU/ AVOIN YLIOPISTO
Teknillinen mekaniikka ja optimointi

23510/02s
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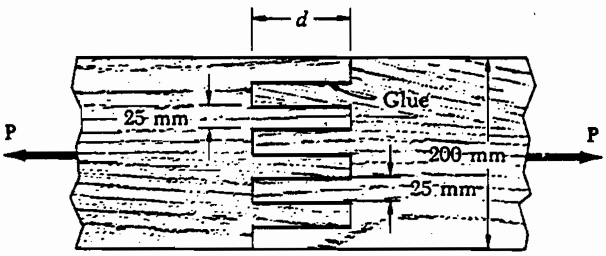
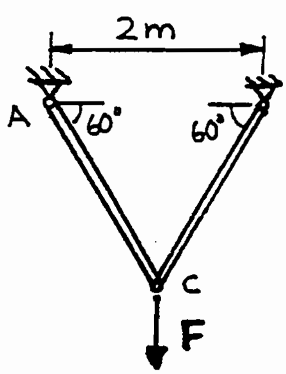
# 23510 Lujuusopin perusteet I

## 1. välikoe 17.10.2002

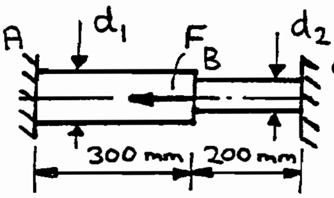
Mukana saa olla vain oma kaavakokoelma (A4 kokoinen lunttilappu molemmin puolin kirjoitetuna)



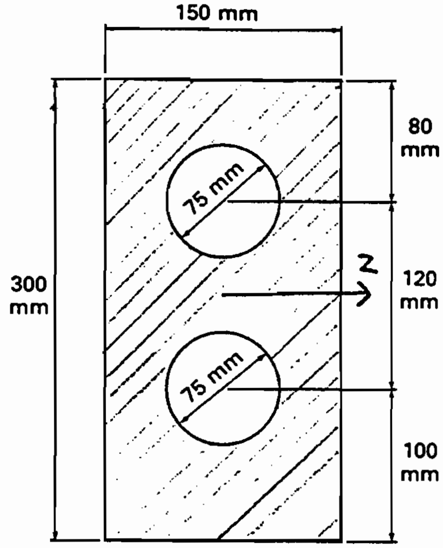

1. Kaksi puusauvaa, paksuus 16 mm ja leveys 200 mm, on liitetty toisiinsa kuvan esittämällä liimaliitoksella. Liiman leikkausmurtolujuus on 900 kPa ja liitosta kuormittaa voima $P = 4\text{ kN}$. Määritä tavittava liitoksen vähimmäispituus $d$, kun varmuusluku on 1,5.



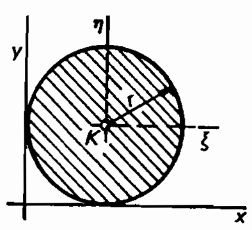

2. Mitoita kuvan ristikon pyörötangosta tehdyt sauvat, kun materiaalin S235RG2 myötöraja on 235 MPa ja voima $F = 15\text{ kN}$. Varmuusluvuksi myötöön nähden halutaan vähintään 1,5. Käytettävissä on pyörötankoja, joiden halkaisijat ovat 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15 ja 16 mm. Laske myös kuinka paljon sauvojen pituudet muuttuvat ja määritä niiden avulla pisteen C siirtymä. $E = 205\text{ GPa}$.




3. Oheista kahdesta pyöreästä terästangosta tehtyä jäykkien tukien väliin kiinnitettyä rakennetta kuormitetaan poikkileikkauksen muutoskohtaan vaikuttavalla voimalla $F = 10\text{ kN}$. $E = 205\text{ GPa}$, $d_1 = 15\text{ mm}$ ja $d_2 = 10\text{ mm}$. Laske rakenteen suurin normaalijännitys sekä kuormituspisteen B siirtymä.




4. Laske kuvan poikkileikkauksen pintakeskiön etäisyys poikkileikkauksen yläreunasta sekä poikkileikkauksen neliömomentti pintakeskiön kautta kulkevan vaakasuoran z-akselin suhteen.




Ympyrä

$A = \pi r^2$

$x_0 = y_0 = r$

$I_x = I_y = \frac{5\pi}{4} r^4$

$I_\xi = I_\eta = \frac{\pi}{4} r^4$

$I_0 = \frac{\pi}{2} r^4$

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**Kaavakokoelma**/*Tapio Salmi* 2001

$\overline{\sigma} = N/A$ $\quad \Delta L = \hat{L} - L$ $\quad \varepsilon = \Delta L / L$ $\quad \hat{\varepsilon} = \Delta L / \hat{L}$

$\nu = -\varepsilon_{\perp}/\varepsilon$ $\quad \Delta V \approx (1-2\nu) V$ $\quad \Delta A \approx -2\nu\varepsilon A$ $\quad \sigma = E\varepsilon$

$\tau = G\gamma$ $\quad E = 2(1+\nu)G$ $\quad$ Teräs: $\sigma_{sall} = R_e / 1{,}5$ $\quad \tau_{sall} = 0{,}6 \cdot \sigma_{sall}$

$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$ $\quad p = \sigma + \tau$ $\quad \sigma = dN/dA$ $\quad \tau = dQ/dA$

$\tau_{yx} = \tau_{xy}$ $\quad \tau_{zx} = \tau_{xz}$ $\quad \tau_{zy} = \tau_{yz}$ $\quad \tau_{max} = \frac{1}{2}(\sigma_I - \sigma_{III})$ $\quad \boldsymbol{u}(P) = \boldsymbol{r}(\hat{P}) - \boldsymbol{r}(P)$

$\boldsymbol{u} = u\,\mathbf{i} + v\,\mathbf{j} + w\,\mathbf{k}$ $\quad \varepsilon_x \approx \frac{\partial u}{\partial x}$ $\quad \varepsilon_y \approx \frac{\partial v}{\partial y}$ $\quad \varepsilon_z \approx \frac{\partial w}{\partial z}$

$\gamma_{xy} \approx \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ $\quad \gamma_{xz} \approx \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$ $\quad \gamma_{yz} \approx \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$ $\quad \varepsilon_{xy} = \frac{1}{2}\gamma_{xy}$

$$[S] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \qquad [V] = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z \end{bmatrix}$$

$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$ $\quad \gamma_{xy} = \frac{1}{G}\tau_{xy}$ $\quad G = \frac{E}{2(1+\nu)}$

$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$ $\quad \gamma_{xz} = \frac{1}{G}\tau_{xz}$

$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$ $\quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$

$\Delta L = \alpha L \Delta T$ $\quad \varepsilon_T = \alpha \Delta T$ $\quad u(x) = \int_{x_0}^{x} \frac{N(x)}{EA}\,dx + u(x_0)$

$\Delta L = \frac{NL}{EA}$ $\quad F = ku$ $\quad k = \frac{EA}{L}$

$\frac{dQ}{dx} = -q(x)$ $\quad \frac{dM_t}{dx} = Q(x)$ $\quad \frac{d^2 M_t}{dx^2} = -q(x)$ $\quad \varepsilon_x = \frac{y}{\rho}$

$\sigma_x = E\varepsilon_x = \frac{Ey}{\rho}$ $\quad \frac{1}{\rho} = \frac{M_t}{EI_z}$ $\quad \sigma_x = \frac{M_t}{I_z} y$ $\quad |\sigma_t|_{max} = \frac{|M_{tz}|}{W_z}$

$W_{z1} = I_z / a_{z1}$ $\quad W_{z2} = I_z / a_{z2}$ $\quad W_z = \min(W_{z1}, W_{z2})$

$\sigma_x = \frac{N}{A} + \frac{M_{tz}}{I_z} y$ $\quad \tau_{xy} = \frac{dR/dx}{b(\bar{y})} = \frac{Q_y S_z(\bar{y})}{I_z b(\bar{y})}$ $\quad \tau_{xy} = 1{,}5\,\bar{\tau}\left[1 - \left(\frac{2y}{h}\right)^2\right]$

$\bar{\tau} = Q/A$ $\quad R_{12} = \frac{S_z(\bar{y})}{I_z}(M_{t2} - M_{t1})$ $\quad \frac{1}{\rho(x)} = |\kappa(x)| = \frac{|v''(x)|}{\left[1 + (v'(x))^2\right]^{3/2}}$

$EI\,v''(x) \approx -M_t(x)$ $\quad EI\,v'''(x) \approx -Q(x)$ $\quad EI\,v''''(x) \approx q(x)$

$\Theta = \frac{d\varphi}{dx}$ $\quad \gamma = r\Theta$ $\quad \tau = G\Theta r$ $\quad \tau_{max} = \frac{1}{2} G\Theta d$

$I_p = \iint_A r^2\,dA$ $\quad I_p = \frac{\pi}{32} d^4$ $\quad T = GI_p\Theta$ $\quad \tau_{max} = \frac{T}{W_v}$

$W_v = I_p / (\frac{1}{2} d)$ $\quad W_v = \frac{\pi}{16} d^3$ $\quad \varphi = \int \frac{T}{G I_p} \, dx + \varphi_0$ $\quad \Delta\varphi = \frac{T L}{G I_v}$

$\tau = \frac{T}{2 A t}$ $\quad \tau_{max} = \frac{T}{W_v}$ $\quad W_v = 2 A t_{min}$ $\quad I_v = \frac{4 A^2}{\oint \frac{ds}{t}}$

$I_v = \frac{4 A^2}{\Sigma (s_i / t_i)}$ $\quad I_v = \alpha b c^3$ $\quad W_v = \beta b c^2$ $\quad I_v \approx \frac{1}{3} b c^3$

$W_v \approx \frac{1}{3} b c^2$ $\quad I_v = \frac{1}{3} \Sigma (s_i t_i^3)$ $\quad W_v = I_v / t_{max}$ $\quad I_v \le I_p$

$I_v \approx \frac{A^4}{40 I_p}$ $\quad v'' + k^2 v = 0$ $\quad v(x) = A \cos(kx) + B \sin(kx)$

$k^2 = P / EI$ $\quad P_n = \frac{\pi^2 EI}{L_n^2}$ $\quad \lambda_n = \frac{L_n}{i}$ $\quad i^2 = I / A$

$\sigma_n = \frac{\pi^2 E}{\lambda_n^2}$ $\quad \lambda_n \ge \pi \sqrt{\frac{E}{\sigma_{-p}}} = \lambda_{nr}$ $\quad \boldsymbol{n}_\alpha = l\,\mathbf{i} + m\,\mathbf{j} + n\,\mathbf{k}$ $\quad l^2 + m^2 + n^2 = 1$

$\boldsymbol{p}_x = \sigma_x \mathbf{i} + \tau_{xy} \mathbf{j} + \tau_{xz} \mathbf{k}$ $\quad \sigma_{-x} = -\sigma_x$ $\quad \boldsymbol{p}_\alpha = l\,\boldsymbol{p}_x + m\,\boldsymbol{p}_y + n\,\boldsymbol{p}_z$

$\boldsymbol{p}_y = \tau_{xy} \mathbf{i} + \sigma_y \mathbf{j} + \tau_{yz} \mathbf{k}$ $\quad \sigma_{-y} = -\sigma_y$ $\quad \sigma_\alpha = \boldsymbol{p}_\alpha \bullet \boldsymbol{n}_\alpha$ $\quad \boldsymbol{\sigma}_\alpha = \sigma_\alpha \boldsymbol{n}_\alpha$

$\boldsymbol{p}_z = \tau_{xz} \mathbf{i} + \tau_{yz} \mathbf{j} + \sigma_z \mathbf{k}$ $\quad \sigma_{-z} = -\sigma_z$ $\quad \boldsymbol{\tau}_\alpha = \boldsymbol{p}_\alpha - \boldsymbol{\sigma}_\alpha$ $\quad \tau_\alpha^2 = p_\alpha^2 - \sigma_\alpha^2$

$\{p_\alpha\} = [S]\{n_\alpha\}$ $\quad \sigma_\alpha = \{n_\alpha\}^T [S] \{n_\alpha\}$ $\quad [S]\{n_{\bar\alpha}\} = \sigma_{\bar\alpha} \{n_{\bar\alpha}\}$

$\det([S] - \sigma_{\bar\alpha}[I]) = 0$ $\quad \sigma_{\bar\alpha}^3 - I_1 \sigma_{\bar\alpha}^2 + I_2 \sigma_{\bar\alpha} - I_3 = 0$ $\quad I_3 = \det[S]$

$I_2 = \begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix} + \begin{vmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & \sigma_z \end{vmatrix} + \begin{vmatrix} \sigma_y & \tau_{yz} \\ \tau_{yz} & \sigma_z \end{vmatrix}$ $\quad I_1 = \sigma_x + \sigma_y + \sigma_z = \mathrm{trace}[S]$

$\sigma_x + \sigma_y + \sigma_z = \sigma_{\bar x} + \sigma_{\bar y} + \sigma_{\bar z}$ $\quad \sigma_I > \sigma_{II} > \sigma_{III}$ $\quad \tau_{max} = \frac{1}{2}(\sigma_I - \sigma_{III})$

$\sigma_{\bar x} = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha$ $\quad \sigma_{\bar x} = \frac{1}{2}(\sigma_x + \sigma_y) + R\cos(2\alpha - 2\varphi)$

$\tau_{\bar x \bar y} = -(\sigma_x - \sigma_y) \sin\alpha \cos\alpha + \tau_{xy}(\cos^2\alpha - \sin^2\alpha)$ $\quad \tau_{\bar x \bar y} = -R \sin(2\alpha - 2\varphi)$

$R = \sqrt{(\frac{1}{2}(\sigma_x - \sigma_y))^2 + \tau_{xy}^2}$ $\quad \tan\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$ $\quad \tau_{xy} \sin(2\varphi) \ge 0$

$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + R \;,\; \alpha = \varphi$ $\quad \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - R \;,\; \alpha = \varphi + \pi/2$

$\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min})$ $\quad \sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\quad R_s = \sigma_{min} / \sigma_{max}$

$\sigma_{Wred} = m \kappa_\sigma \sigma_W$ $\quad K_t = \sigma_{max} / \sigma_{nim}$ $\quad K_f = \sigma_{Wsileä} / \sigma_{Wlovi}$

$K_f = 1 + q(K_t - 1)$ $\quad I_z = \iint_A y^2 \, dA$ $\quad I_z = I_\zeta + A y_0^2$

$I_{yz} = \iint_A yz \, dA$ $\quad I_{yz} = I_{\eta\zeta} + A y_0 z_0$

$I_z = \frac{1}{12} b h^3$ (suorakulmio) $\quad I_z = \frac{\pi}{4} r^4$ (ympyrä) $\quad I_p = \frac{\pi}{2} r^4$ (ympyrä)